Simulating Language 2021, week 3 pre-reading questions, part 1 answers

The first few questions are on the basics of probabilities.

1. **The weather forecaster tells me that tomorrow there is a 25% chance of rain. Assuming they are an accurate forecaster, what is the probability that it will rain tomorrow?**

The correct answer is 0.25. Probabilities sum to 1: probability 1 means something will definitely happen (or it will always happen, if you prefer to think about it that way), probability 0 means something will definitely not happen / never happen, probability 0.25 means it will happen about a quarter of the time, which is the same as saying there is a 25% chance of it happening.

2. What is the probability that it will *not* rain tomorrow?

The correct answer is 0.75: the probability that it will not rain is 1 minus the probability that it will rain, which is 1 - 0.25, which is 0.75. We know for sure it will either rain or not rain tomorrow, so the probability of those two events taken together must sum to 1 (i.e. it will *definitely* either rain or not rain tomorrow). We already know that the probability of it raining is 0.25, so the remainder of the probability must be on the other event, that it does not rain.

3. Which of the following is not a probability?

There aren't probabilities: -0.1, 1.1. Probabilities must like between 0 (something will never happen) or 1 (something will always happen), so values lower than 0 or higher than 1 are not probabilities.

The next bunch of questions are on likelihoods: the probability of some event happening given that some state of affairs is true.

4. Consider an unbiased, 'fair' dice*. What is the probability that any roll of the dice will produce a 1? Note: in the notation used in the reading, we would write this probability as something like p(1 l fair-dice), i.e. the probability of getting a 1 given that we are rolling the fair dice.

*** For grammar puritans: consider an unbiased, 'fair' die.**
*** For D&Ders: it's a D6.**

The correct answer here is 1/6. There are 6 possible outcomes for this dice: it will roll a 1, it will roll a 2, etc etc, up to it will roll a 6. Since the dice is fair, each of those outcomes is equally likely, so each gets an equal share of the available probability. Since we know that every time we roll a dice it will come up as *something* (i.e. it won't roll forever), we have a probability mass of 1 to be divided up between the 6 possible outcomes (rolling a 1, a 2, etc). That means each outcome has a probability of 1 divided by 6, or 1/6, or 1 in 6.

5. What is the probability that it will roll a 6?

The same answer, 1/6, for the same reason. Even though rolling a 6 is kind of exciting, if the dice is fair it's no more or less likely than rolling a 1.

6. What is the probability that it will roll a 3?
The same answer, 1/6, for the same reason.

7 What is the probability that it will roll a number higher than 3 (i.e. 4, 5 or 6)?
The correct answer is 1/2. There are a couple of ways you can arrive at this answer.

One would be to note that the dice, when viewed from this perspective, can do two things: it can roll a number less than or equal to 3, or it can roll a number higher than 3. There are 3 ways it can come up as less than or equal to 3 (you can roll a 1, 2 or 3), and 3 ways it can come up as higher than 3 (you can roll a 4, 5 or 6). So there are two possibilities, which seem equally likely, therefore they each have probability 1/2.

Another way to do it would be to sum the individual probabilities of each dice roll. We already know that the probability of rolling a 1 is 1/6. The probability of rolling a 2 is 1/6. The probability of rolling a 3 is 1/6. So the probability of rolling a 1, 2 or 3 is 1/6 + 1/6 + 1/6, which is 3/6, which is the same as 1/2: we *sum the probabilities* if we are interested in whether one event *or* another event will happen (assuming those events are mutually exclusive, which they are here: the dice can't simultaneously come up as a 1 and a 2). Similarly, the probability of rolling a 4 *or* a 5 *or* a 6 is 1/6 + 1/6 + 1/6 = 3/6 = 1/2.

8. How about if we roll the dice twice? What is the probability that it will produce a 6 then a 6?

The correct answer is 1/36.

In answering question 7, we were interested in situations where one of several possible events occurred: either the dice will roll a 1 or it will roll a 2 or it will roll a 3, etc - there we added probabilities. Here we are interested in sequences or combinations of independent events, in which case we *multiply* the probabilities. The probability of rolling a 6 on a single roll is 1/6. So the probability of rolling a 6 *then* rolling a 6 is 1/6 * 1/6, which is 1/36.

If you don't believe me about multiplying probabilities, try this: write down all the possible combinations of what you get on the first roll then what you get on the second roll. This gives you:

1 then 1
1 then 2
1 then 3
1 then 4
1 then 5
1 then 6
2 then 1
2 then 2
...
2 then 6
3 then 1
...

3 then 6
...
6 then 6

If you do this, you will notice that there are 36 possible outcomes of rolling a dice twice. Only one of then is the desired “6 then 6” combination, and since the dice is fair, every one of these 36 outcomes is equally probable. The probability of rolling a 6 then a 6 must therefore be 1 in 36. If you get confused when you are manipulating probabilities, it can often be helpful to enumerate the possible outcomes in this way, i find it helps me be clear about what I am doing.

9. A 3 then a 5?

Again, the correct answer is 1/36, for the same reasons.

10. A number higher than 3 on both rolls?
The correct answer is 1/4. We already worked out that the probability of rolling a number higher than 3 is 1/2. So the probability of doing it twice in a row is 1/2 * 1/2 = 1/4. Or if you prefer enumeration, you can use the exhaustive list of 36 combinations you wrote down for question 8, and verify that 18 of those 36 possibilities involve two rolls greater than 3: 18/36 = 1/2.